$M = 10^4$, $N_2 = \{[9 \times 1 + 10]9 + 10^2\}9 + 10^3$, or $9^3 + 9^2 \cdot 10 + 9 \cdot 10^2 + 10^3$.

$M = 10^5$, $N_2 = 9^4 + 9^3 \cdot 10 + 9^2 \cdot 10^2 + 9 \cdot 10^3 + 10^4$.

. . . . . .

$M = 10^t$, $N_2 = 9^{t-1} + 9^{t-2} \cdot 10 + 9^{t-3}10^2 + \cdots + 9 \cdot 10^{t-2} + 10^{t-1}$.

$M = 10^{t+1}$, $N_2 = 9^t + 9^{t-1} \cdot 10 + 9^{t-2}10^2 + \cdots + 9^2 \cdot 10^{t-2} + 9 \cdot 10^{t-1} + 10^t$.

In general, for $M = 10^t$, $N_2 = (10^t - 9^t)/(10 - 9) = 10^t - 9^t$; $N_1 = 9^t$.

$$\operatorname*{Lim}_{t \doteq \infty} \frac{N_1}{N_2} = \lim_{t \doteq \infty} \left[\frac{9^t}{10^t - 9^t} \right] = 0.$$

For the more general values of M such as $a_0 10^s + a_1 10^{s-1} + \cdots + a_{s-1}10 + a_s$, wherein none of the a's is equal to 9,

$$N_2 = a_0(10^s - 9^s) + a_1(10^{s-1} - 9^{s-1}) \cdots + a_{s-1}(10 - 9),$$

while

$$N_1 = a_0 9^s + a_1 9^{s-1} + \cdots + a_{s-1}9 + a_s.$$

Hence,

$$\lim_{s \doteq \infty} \text{ of } \frac{N_1}{N_2} = 0.$$

If a_3 should be the first a which is 9, then

$$N_2 = a_0(10^s - 9^s) + a_1(10^{s-1} - 9^{s-1}) + a_2(10^{s-2} - 9^{s-2}) + a_3(10^{s-3} - 9^{s-3}) + a_4 10^{s-4} + \cdots + a_{s-1}10 + a_s + 1,$$

while

$$N_1 = a_0 9^s + a_1 9^{s-1} + a_2 9^{s-2} + a_3 9^{s-3} - 1.$$

Hence,

$$\lim_{s \doteq \infty} \frac{N_1}{N_2} = 0.$$

With very slight changes the formula may be made to apply to the corresponding cases for 0, 1, 2, $\cdots$ 8.

MISCELLANEOUS QUESTIONS.

EDITED BY R. D. CARMICHAEL.

Editorial Note.—Several questions of interest are still outstanding. Question 4 is important for teaching analytic geometry (see December, 1913, issue). All efforts to increase the number of concrete applications available are to be commended.

Question 6 (February, 1914, issue) is of great significance in view of the present strong tendency toward increasing the vocational training in the schools.

Question 8 (March, 1914, issue) has an important bearing upon the relation of the colleges to the schools of education. Should not professional training for

teachers include practice teaching? In some cases only *observation* is expected, in others the practice teachers are put in actual charge of classes for a semester and given full responsibility under supervision. It has long been recognized that the training of teachers for elementary schools should include practice teaching; there is now at least one state college for the preparation of high school teachers where the responsible practice teaching is an important feature; why should not some sort of practice teaching be required of advanced students who are preparing for college positions?

Question **9** (March issue), so far as it pertains to coördinated courses in college mathematics, has just acquired a fresh interest in the new book by Professor Slichter, of the University of Wisconsin, entitled "Elementary Mathematical Analysis," which is designed to provide work equivalent to the traditional courses in trigonometry, college algebra, and analytic geometry. This book will be reviewed in the MONTHLY at an early date. Meanwhile, contributions concerning the general question of coördinated collegiate mathematics are desired.

Questions **11** to **15** (issues of May, June, September, and October) are of more limited interest, but nevertheless represent precisely the attitude of mind that this department is striving to foster, namely, a real question about a real difficulty encountered, or a genuine desire to gain further light upon important matters connected with the teacher's work.

Question **13** is possibly an exception to this last statement in that it seems to be an inquiry as to what is going on in the world outside rather than an expression of inner conflict of opinion. But, even so, it is a most worthy question. The teacher must look to the world outside for inspiration and courage to escape from submersion in the quiet waters of the daily routine. Professor Lehmer in California was surprised to find that Professor Bussey in Minnesota did not know about his course in synthetic projective geometry which he had been giving to junior college students for ten years. But how can we know about these things unless we *communicate* them to others? And how can we communicate them without a *medium* of communication? This is the mission of the MONTHLY and especially of the department of "Miscellaneous Questions." If every one who has something good which he has been concealing will come forward and let his light shine for the benefit of others, there will be a general awakening all along the line.

NEW QUESTIONS.

16. To what extent should a first course in geometry be made to appeal to a student's intuition? Should the subject be presented in a manner so as to depend to the greatest possible degree upon his previous experience, or is it desirable to attempt to make it more abstract and formal?

17. In analytic geometry, simplicity and directness are gained by making the condition for the collinearity of three points and the equation of the straight line depend upon the determinant formula for the area of a triangle. Similar advantages are gained by making the condition of complanarity of four points and the equation of the plane depend upon the determinant formula for the volume of a tetrahedron. The former is given in the texts. Why should not the latter be given? A uniform method of developing these two determinants is desired from some contributor.

18. In view of the present pressure for saving time and gaining efficiency, what are the most important sources of economy in the mathematical courses of the high school and the first two years in college?